Published by Institute of Physics Publishing for SISSA

Received: *August 3, 2005*
Revised: *November 15, 2005*
Accepted: *January 17, 2006*
Published: *January 24, 2006*

Non-commutative multi-dimensional cosmology

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Abstract: A non-commutative multi-dimensional cosmological model is introduced and used to address the issues of compactification and stabilization of extra dimensions and the cosmological constant problem. We show that in such a scenario these problems find natural solutions in a universe described by an increasing time parameter.

Keywords: Classical Theories of Gravity, Non-Commutative Geometry.

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Contents

1. Introduction

Non-commutativity between space-time coordinates, first introduced in [1], has been attracting considerable attention in the recent past [2–4]. This renewed interest has its roots in the development of string and M-theories, [5, 6]. However, in all fairness, investigation of non-commutative theories may also be justified in its own right because of the interesting predictions regarding, for example, the IR/UV mixing and non-locality [7], Lorentz violation [8] and new physics at very short distance scales [9–11]. The impact of non-commutativity in cosmology has also been considerable and has been addressed in different works [12]. Hopefully, non-commutative cosmology would lead us to the formulation of semiclassical approximations of quantum gravity and tackles the cosmological constant problem [13]. Of particular interest would be the application of non-commutativity to multi-dimensional cosmology.

Multi-dimensional spaces were introduced for the geometric unification of interactions by Kaluza and Klein and have since been a source of inspiration for numerous authors [14]. Also, the introduction of extra dimensions suggests possible solutions to some of the important problems that cosmology has been facing, namely, the mass hierarchy and cosmological constant problem, to name but a few. That said, it should also be mentioned that the question of compactification and the stabilization of the extra dimensions is a challenge one cannot avoid. In this paper, we have considered a multi-dimensional cosmology with a cosmological constant and introduced non-commutativity between the scale factors of our ordinary space and the extra dimensions. We have shown that the classical cosmology of this model can be solved exactly. These solutions offer an explanation for the cosmological constant problem and show how the extra dimensions can be compactified.

2. The model

We consider a cosmological model in which the space-time is assumed to be of FRW type with a d-dimensional internal space. The corresponding metric can be written as

$$ds^2 = -dt^2 + \frac{R^2(t)}{\left(1+\frac{k}{4}r^2\right)^2}(dr^2 + r^2 d\Omega^2) + a^2(t) g_{ij}^{(d)} dx^i dx^j \ , \tag{2.1}$$

where the total number of dimensions is $D = 4 + d$, $k = 1, 0, -1$ represents the usual spatial curvature, $R(t)$ and $a(t)$ are the scale factors of the universe and the radius of the d-dimensional internal space respectively and $g_{ij}^{(d)}$ is the metric associated with the internal space, assumed to be Ricci-flat. The Ricci scalar corresponding to metric (2.1) is

$$\mathcal{R} = 6\left[\frac{\ddot{R}}{R} + \frac{k+\dot{R}^2}{R^2}\right] + 2\,d\frac{\ddot{a}}{a} + d(d-1)\left(\frac{\dot{a}}{a}\right)^2 + 6\,d\frac{\dot{a}\dot{R}}{aR}, \tag{2.2}$$

where a dot represents differentiation with respect to t. Let a_0 be the compactification scale of the internal space at present time and

$$v_d \equiv v_0 \times v_i \equiv a_0^d \times \int_{M_d} d^d x \sqrt{-g^{(d)}}, \tag{2.3}$$

the corresponding total volume of the internal space. Substitution of equation (2.2) and use of definition (2.3) in the Einstein-Hilbert action functional with a D-dimensional cosmological constant Λ

$$\mathcal{S} = \frac{1}{2k_D^2}\int_M d^D x \sqrt{-g}(\mathcal{R} - 2\Lambda) + \mathcal{S}_{YGH}, \tag{2.4}$$

where k_D is the D-dimensional gravitational constant and $\mathcal{S}_{YGH}$ is the York-Gibbons-Hawking boundary term, leads to, after dimensional reduction

$$\mathcal{S} = -v_{D-1}\int dt \left\{6\dot{R}^2\Phi R + 6R\dot{R}\dot{\Phi}R^2 + \frac{d-1}{d}\frac{\dot{\Phi}^2}{\Phi}R^3 - 6k\Phi R - 2\Phi R^3\Lambda\right\}, \tag{2.5}$$

where

$$\Phi = \left(\frac{a}{a_0}\right)^d, \tag{2.6}$$

and we have set $v_{D-1} = 1$. To make the lagrangian manageable, consider the following change of variables

$$\Phi R^3 = \Upsilon^2(x_1^2 - x_2^2), \tag{2.7}$$

where $R = R(x_1, x_2)$ and $\Phi = \Phi(x_1, x_2)$ are functions of new variables x_1, x_2. Let

$$\begin{cases} \Phi^{\rho_+} R^{\sigma_-} = \Upsilon(x_1 + x_2), \\ \\ \Phi^{\rho_-} R^{\sigma_+} = \Upsilon(x_1 - x_2), \end{cases} \tag{2.8}$$

such that for $d \neq 3$, $\Upsilon = 1$ and we have

$$\begin{cases} \rho_\pm = \frac{1}{2} \pm \frac{3}{4}\sqrt{\frac{d+2}{3d}} \mp \frac{1}{4\sqrt{\frac{d+2}{3d}}}, \\ \sigma_\pm = \frac{1}{2}\left(3 \mp \frac{1}{\sqrt{\frac{d+2}{3d}}}\right), \end{cases} \tag{2.9}$$

while for $d = 3$, $\Upsilon = \frac{3}{\sqrt{5}}$ and

$$\begin{cases} \rho_\pm = \frac{1}{2} \pm \frac{\sqrt{5}}{10}, \\ \sigma_\pm = 3\left(\frac{1}{2} \pm \frac{\sqrt{5}}{10}\right). \end{cases} \tag{2.10}$$

Using the above transformations introduced in [15] and concentrating on $k = 0$, the lagrangian becomes

$$\mathcal{L} = -4\left(\frac{d+2}{d+3}\right)\left\{\dot{x_1}^2 - \dot{x_2}^2 - \frac{\Lambda}{2}\left(\frac{d+3}{d+2}\right)\left(x_1^2 - x_2^2\right)\right\}. \tag{2.11}$$

Up to an overall constant coefficient, we can write the effective hamiltonian as

$$\mathcal{H} = \frac{p_1^2}{4} - \frac{p_2^2}{4} + \omega^2\left(x_1^2 - x_2^2\right), \tag{2.12}$$

where ω^2 is

$$\omega^2 = \frac{1}{2}\left(\frac{d+3}{d+2}\right)\Lambda. \tag{2.13}$$

The role of the variables x_1 and x_2 can be grasped easily if one multiplies equation (2.7) by Λ. Equations (2.12) and (2.13) then show that the potential energy for our system of harmonic oscillators is proportional to the vacuum energy Λ times the volume of the multidimensional universe. Also, as has been discussed in [16], the wrong sign in the hamiltonian for the x_2 component is no cause for concern since the equations of motion resulting from the hamiltonian are similar to those describing a system of two ordinary uncoupled harmonic oscillators. In other words, the potential in equation (2.12) is given by $V = \omega^2(x_1^2 - x_2^2)$ so that the equations of motion become

$$\ddot{x_1} = -\frac{\partial V}{\partial x_1} \quad \text{and} \quad \ddot{x_2} = \frac{\partial V}{\partial x_2}. \tag{2.14}$$

For the x_1 component, the force is given by minus the gradient of the potential, whilst for the x_2 component the force is given by plus the gradient of the potential. Therefore, the criteria for the stability of motion for the x_2 degree of freedom is that the potential has to have a maximum in the (x_2, V) plane. This is just opposite to the case of the x_1 degree of freedom where stability requires a minimum for the potential in the (x_1, V) plane.

3. Classical solutions

3.1 Commutative case

Let us start by supposing that the dynamical variables defined in (2.8) and their conjugate momenta satisfy [16]

$$\{x_\mu, p_\nu\}_P = \eta_{\mu\nu}, \tag{3.1}$$

where $\eta_{\mu\nu}$ is the two dimensional Minkowski metric and $\{,\}_P$ represents the Poisson bracket. In view of the above, hamiltonian (2.12) can be written as

$$\mathcal{H} = \frac{1}{4}\eta^{\mu\nu}p_\mu p_\nu + \omega^2\eta^{\mu\nu}x_\mu x_\nu, \tag{3.2}$$

and equations of motion become

$$\begin{cases} \dot{x_\mu} = \{x_\mu, \mathcal{H}\}_P = \frac{1}{2}p_\mu, \\ \\ \dot{p_\mu} = \{p_\mu, \mathcal{H}\}_P = -2\omega^2 x_\mu, \end{cases} \tag{3.3}$$

where in obtaining these equations we have used relations (3.1). Now, using equations (3.3) we obtain

$$\ddot{x_\mu} + \omega^2 x_\mu = 0. \tag{3.4}$$

The solution of the above equations reads

$$x_\mu(t) = A_\mu e^{i\omega t} + B_\mu e^{-i\omega t}, \tag{3.5}$$

where A_μ and B_μ are constants of integration. We note that hamiltonian constraint ($\mathcal{H} = 0$) imposes the following relation on these constants

$$A_\mu B^\mu = 0. \tag{3.6}$$

Finally, using equations (2.8) and (2.6), the scale factors take on the following forms for $d \neq 3$

$$\begin{cases} a(t) = k_1 \left[\sin(\omega t + \phi_1)\right]^{\frac{-2(\sigma_+)\sqrt{\frac{d+2}{3d}}}{d-3}} \left[\sin(\omega t + \phi_2)\right]^{\frac{2(\sigma_-)\sqrt{\frac{d+2}{3d}}}{d-3}}, \\ \\ R(t) = k_2 \left[\sin(\omega t + \phi_1)\right]^{\frac{2(\rho_-)d\sqrt{\frac{d+2}{3d}}}{d-3}} \left[\sin(\omega t + \phi_2)\right]^{\frac{-2(\rho_+)d\sqrt{\frac{d+2}{3d}}}{d-3}}, \end{cases} \tag{3.7}$$

where k_1 and k_2 are arbitrary constants and ϕ_1 and ϕ_2 are arbitrary phases. Note that if ω^2 is negative, the trigonometric functions are replaced by their hyperbolic counterparts in the above solutions. The sign of ω^2 relates to the sign of the cosmological constant (2.13). For $d = 3$, similar solutions are easily found from transformations (2.10).

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3.2 Non-commutative case

We now concentrate on the non-commutativity concepts with Moyal product in phase space. The Moyal product in phase space may be traced to an early intuition by Wigner [17] which has been developing over the past decades [18]. Non-commutativity in classical physics [19] is described by the Moyal product law between two arbitrary functions of position and momenta as

$$(f \star_\alpha g)(x) = \exp\left[\frac{1}{2}\alpha^{ab}\partial_a^{(1)}\partial_b^{(2)}\right] f(x_1)g(x_2)|_{x_1=x_2=x}, \tag{3.8}$$

such that

$$\alpha_{ab} = \begin{pmatrix} \theta_{\mu\nu} & \eta_{\mu\nu}+\sigma_{\mu\nu} \\ -\eta_{\mu\nu}-\sigma_{\mu\nu} & \beta_{\mu\nu} \end{pmatrix}, \tag{3.9}$$

where the $N \times N$ matrices θ and β are assumed to be antisymmetric with $2N$ being the dimension of the classical phase space and σ can be written as a combination of θ and β. With this product law, the deformed Poisson brackets can be written as

$$\{f, g\}_\alpha = f \star_\alpha g - g \star_\alpha f. \tag{3.10}$$

A simple calculation shows that

$$\begin{aligned} \{x_\mu, x_\nu\}_\alpha &= \theta_{\mu\nu}, \\ \{x_\mu, p_\nu\}_\alpha &= \eta_{\mu\nu} + \sigma_{\mu\nu}, \\ \{p_\mu, p_\nu\}_\alpha &= \beta_{\mu\nu}\ . \end{aligned} \tag{3.11}$$

It is worth noticing at this stage that in addition to non-commutativity in (x_1, x_2) we have also considered non-commutativity in the corresponding momenta. This should be interesting since its existence is in fact due essentially to the existence of non-commutativity on the space sector [18, 19] and it would somehow be natural to include it in our considerations.

Now, consider the following transformation on the classical phase space (x_μ, p_μ)

$$\begin{aligned} x'_\mu &= x_\mu - \frac{1}{2}\theta_{\mu\nu}p^\nu, \\ p'_\mu &= p_\mu + \frac{1}{2}\beta_{\mu\nu}x^\nu. \end{aligned} \tag{3.12}$$

It can easily be checked that if (x_μ, p_μ) obey the usual Poisson algebra (3.1), then

$$\begin{aligned} \{x'_\mu, x'_\nu\}_P &= \theta_{\mu\nu}, \\ \{x'_\mu, p'_\nu\}_P &= \eta_{\mu\nu} + \sigma_{\mu\nu}, \\ \{p'_\mu, p'_\nu\}_P &= \beta_{\mu\nu}. \end{aligned} \tag{3.13}$$

These commutation relations are the same as (3.11). Consequently, for introducing non-commutativity, it is more convenient to work with Poisson brackets (3.13) than α-star deformed Poisson brackets (3.11). It is important to note that the relations represented by equations (3.11) are defined in the spirit of the Moyal product given above. However, in the relations defined in (3.13), the variables (x_μ, p_μ) obey the usual Poisson bracket relations so that the two sets of deformed and ordinary Poisson brackets represented by relations (3.11) and (3.13) should be considered as distinct.

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Let us change the commutative hamiltonian (3.2) with minimal variation to

$$\mathcal{H}' = \frac{1}{4}\eta^{\mu\nu}p'_\mu p'_\nu + \omega^2\eta^{\mu\nu}x'_\mu x'_\nu, \tag{3.14}$$

where we have the commutation relations

$$\begin{aligned} \{x'_\mu, x'_\nu\}_P &= \theta\epsilon_{\mu\nu}, \\ \{x'_\mu, p'_\nu\}_P &= (1+\sigma)\eta_{\mu\nu}, \\ \{p'_\mu, p'_\nu\}_P &= \beta\epsilon_{\mu\nu}, \end{aligned} \tag{3.15}$$

with $\epsilon_{\mu\nu}$ being a totally anti-symmetric tensor and σ is given by

$$\sigma = \frac{1}{4}\beta\theta. \tag{3.16}$$

We have also set $\theta_{\mu\nu} = \theta\epsilon_{\mu\nu}$ and $\beta_{\mu\nu} = \beta\epsilon_{\mu\nu}$. Using the transformation introduced in (3.12), hamiltonian (3.14) becomes

$$\mathcal{H} = \frac{1}{4}\left(1-\omega^2\theta^2\right)\eta^{\mu\nu}p_\mu p_\nu + \left(\omega^2 - \frac{\beta^2}{16}\right)\eta^{\mu\nu}x_\mu x_\nu - \left(\frac{\beta}{4} + \theta\omega^2\right)\epsilon^{\mu\nu}x_\mu p_\nu. \tag{3.17}$$

The equations of motion corresponding to hamiltonian (3.17) are

$$\begin{cases} \dot{x_\mu} = \{x_\mu, \mathcal{H}\}_P = \frac{1}{2}\left(1-\omega^2\theta^2\right)p_\mu + \left(\frac{\beta}{4} + \theta\omega^2\right)\epsilon_{\mu\nu}x^\nu, \\ \\ \dot{p_\mu} = \{p_\mu, \mathcal{H}\}_P = -2\left(\omega^2 - \frac{\beta^2}{16}\right)x_\mu + \left(\frac{\beta}{4} + \theta\omega^2\right)\epsilon_{\mu\nu}p^\nu, \end{cases} \tag{3.18}$$

where we have used relations (3.1). It can again be easily checked that if one writes the equations of motion for non-commutative variables, equations (3.15), with respect to hamiltonian (3.14) and uses transformation rules (3.12), one gets a linear combination of the equations of motion (3.18). This points to the fact that these two approaches are equivalent. Now, as a consequence of equations of motion (3.18), one has

$$\ddot{x}_\mu - 2\left(\frac{\beta}{4} + \theta\omega^2\right)\epsilon_{\mu\nu}\dot{x}^\nu + \left[\left(1-\omega^2\theta^2\right)\left(\omega^2 - \frac{\beta^2}{16}\right) + \left(\frac{\beta}{4} + \theta\omega^2\right)^2\right]x_\mu = 0. \tag{3.19}$$

Note that upon setting $\theta = \beta = 0$, we get back equation (3.4). Here, the equations have an additional velocity term which may be interpreted as viscosity. The solution to equation (3.19) can be written as follows

$$\begin{cases} x_1(t) = Ae^{(-\alpha+i\gamma)t} + Be^{(-\alpha-i\gamma)t} + Ce^{(\alpha+i\gamma)t} + De^{(\alpha-i\gamma)t}, \\ \\ x_2(t) = Ae^{(-\alpha+i\gamma)t} + Be^{(-\alpha-i\gamma)t} - Ce^{(\alpha+i\gamma)t} - De^{(\alpha-i\gamma)t}, \end{cases} \tag{3.20}$$

where

$$\alpha = \left(\theta\omega^2 + \frac{\beta}{4}\right) \quad \text{and} \quad \gamma = \left[\left(1-\omega^2\theta^2\right)\left(\omega^2 - \frac{\beta^2}{16}\right)\right]^{1/2}, \tag{3.21}$$

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with A, B, C and D being the constants of integration. The hamiltonian constraint, $\mathcal{H} = 0$, leads to

$$[\gamma + i\alpha]AD + [\gamma - i\alpha]BC = 0. \tag{3.22}$$

Setting $\theta = \beta = 0$ in (3.22) reproduces relation (3.6) in the commutative case, as one would expect. Also, from equations (2.8) and (2.6) we recover the scale factors as ($d \neq 3$)

$$\begin{cases} a(t) = k_1 \left[\sin(\gamma t + \phi_1)\right]^{\frac{-2(\sigma_+)\sqrt{\frac{d+2}{3d}}}{d-3}} \left[\sin(\gamma t + \phi_2)\right]^{\frac{2(\sigma_-)\sqrt{\frac{d+2}{3d}}}{d-3}} \exp\left[\dfrac{(6\alpha)\sqrt{\frac{d+2}{3d}}}{d-3} t\right], \\ \\ R(t) = k_2 \left[\sin(\gamma t + \phi_1)\right]^{\frac{2(\rho_-)d\sqrt{\frac{d+2}{3d}}}{d-3}} \left[\sin(\gamma t + \phi_2)\right]^{\frac{-2(\rho_+)d\sqrt{\frac{d+2}{3d}}}{d-3}} \exp\left[\dfrac{-(2d\alpha)\sqrt{\frac{d+2}{3d}}}{d-3} t\right], \end{cases} \tag{3.23}$$

where, k_1 and k_2 are constants, α and γ are defined in (3.21) and ϕ_1 and ϕ_2 are arbitrary phases. Note that in the commutative case the exponential terms were not present. Also, it is worth noticing that the first equation in (3.23) shows that if $\frac{\alpha}{d-3}$ is negative then the scale factor corresponding to the extra dimensions compactifies to zero as time increases. Again, as in the previous case, if γ^2 is negative, then in the above solutions hyperbolic functions would replace the corresponding trigonometric ones. In this case the above scale factors for late times become

$$\begin{cases} a(t) = k_1 e^{\frac{-2(\gamma\sigma_+)\sqrt{\frac{d+2}{3d}}}{d-3} t} e^{\frac{2(\gamma\sigma_-)\sqrt{\frac{d+2}{3d}}}{d-3} t} e^{\frac{(6\alpha)\sqrt{\frac{d+2}{3d}}}{d-3} t}, \\ \\ R(t) = k_2 e^{\frac{2(\gamma\rho_-)d\sqrt{\frac{d+2}{3d}}}{d-3} t} e^{\frac{-2(\gamma\rho_+)d\sqrt{\frac{d+2}{3d}}}{d-3} t} e^{\frac{-(2d\alpha)\sqrt{\frac{d+2}{3d}}}{d-3} t}. \end{cases} \qquad t \rightarrow +\infty \tag{3.24}$$

To make $a(t)$ stabilized, the total exponential in the first equation should become finite. To this end we put the sum of the exponents equal to zero. One thus finds a relation between α and γ as

$$\alpha = \frac{\gamma}{3}(\sigma_+ - \sigma_-). \tag{3.25}$$

This equation can now be used for choosing (tuning) the scale factors such that $a(t)$ approaches a finite value for large t while the scale factor representing the universe, $R(t)$, grows exponentially. This points to the stabilization of extra dimensions and can also be interpreted as their compactification relative to the scale factor of the universe, see figure 1. Note that for $d = 3$, the relation (3.25) also holds.

At this point it would be appropriate to address the cosmological constant problem, the huge disparity between the values of the cosmological constant predicted in cosmology and particle physics. A number of approaches have been suggested to address this problem [20]. However, as will be shown below, this problem has a natural solution in the non-commutative approach. We note from above that ω^2 is proportional to the cosmological constant Λ. Comparing equations (2.13), (3.7), (3.21) and (3.23) we propose that the relation (2.13) between the oscillating frequency γ and the cosmological constant Λ_{nc}

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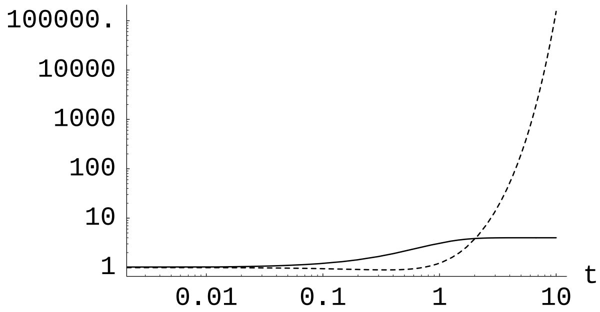

Figure 1: Scale factor of the extra dimensions, solid line and that of the ordinary space, dotted line. The values of α and γ are such that equation (3.25) is satisfied and $d = 1$.

continues to hold, that is

$$\Lambda_{nc} = 2\left(\frac{d+2}{d+3}\right)\gamma^2 = 2\left(\frac{d+2}{d+3}\right)\left(1-\omega^2\theta^2\right)\left(\omega^2 - \frac{\beta^2}{16}\right). \tag{3.26}$$

As has already been noticed, the non-commutativity concept in our model is closely related to extra dimensions and will disappear if the extra dimensions disappear, that is if we set $d = 0$. Since the appearance of extra dimensional effects could be more naturally attributed to short distance scales, it would be reasonable to assume that Λ_{nc} is the cosmological constant associated with particle physics. Now, from equations (3.26) and (2.13) one has

$$\Lambda_{nc} = \left[1 - \frac{1}{2}\left(\frac{d+3}{d+2}\right)\theta^2\Lambda\right]\left[\Lambda - \frac{1}{8}\left(\frac{d+2}{d+3}\right)\beta^2\right]. \tag{3.27}$$

Since the cosmological constant Λ appearing in the above equation is that representing large scales and has consequently a very small value, we see from equation (3.27) that the particle physics cosmological constant is

$$\Lambda_{nc} \sim -\frac{1}{8}\left(\frac{d+2}{d+3}\right)\beta^2. \tag{3.28}$$

If β^2 is large relative to Λ then from (3.28) Λ_{nc} will become much larger than Λ. The above analysis points to a possible solution of the long standing cosmological constant problem. The above argument is also true up to a multiplicative constant for the case $d = 3$.

To have a geometrical understanding of non-commutativity of the scale factors, let us see how such a relation looks. One may show that for $d \neq 3$

$$\{\Phi, R\}_\alpha = \frac{4d\left(\frac{d+2}{3d}\right)^{1/2}}{d-3}\frac{\theta}{R^2}, \tag{3.29}$$

while for $d = 3$

$$\{\Phi, R\}_\alpha = -\frac{5\sqrt{5}}{216}\frac{\theta}{R^2}. \tag{3.30}$$

These relations show that, no matter how large θ may get, the effect of non-commutativity would be very small in the present epoch due to the large value of the scale factor R.

4. Conclusions

In this paper we have introduced non-commutativity between scale factors of the ordinary universe and extra dimensions in a multi-dimensional cosmological model. We have shown that the classical solutions of such a model clearly point to a possible resolution of the cosmological constant problem and compactification of extra dimensions. We have also shown that in this model the two parameters representing non-commutativity are not independent and satisfy a certain relationship, equation (3.25), which may be used for the purpose of fine-tuning so as to make the extra dimensions stabilized.

Acknowledgments

The authors would like to thank the anonymous referee for valuable comments.

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